

# Ages of old objects constraints on cosmic opacity

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## Abstract

In this paper, we use the ages of 32 old passive galaxies distributed over the redshift interval  $0.11 < z < 1.84$  to obtain opacity free luminosity distances and put limits on cosmic opacity by comparing them with the 580 distance moduli of type Ia supernovae from the so-called Union 2.1 compilation. The cosmic opacity is parametrized by  $\tau(z) = 2\epsilon z$  and  $\tau(z) = \epsilon z/(1+z)$  (for  $\epsilon = 0$  the transparent universe is recovered). We find  $\epsilon < 0.094$  from the first parametrization and  $\epsilon < 0.07$  from the second parametrization, both limits at 95% c.l. In addition, by considering the total age as inferred from Planck (2015) analysis, we find the incubation time  $t_{inc} = 1.66 \pm 0.29$  Gyr and  $t_{inc} = 1.23 \pm 0.27$  Gyr at 68% c.l. for each parametrization, respectively.

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## I. INTRODUCTION

Since 1998, type Ia supernovae (SNe Ia) observations (Riess *et al.* 1998, Perlmutter *et al.* 1998, Suzuki *et al.* 2012, Bertoule *et al.* 2014) have been an important tool to access the current cosmic acceleration and test different cosmological models. In order to explain cosmic acceleration, if one wants to preserve Einstein's General Relativity Equations together with spacetime isotropy and homogeneity, one has to postulate some source of negative pressure. The sources of negative pressure that have been hypothesized include Einstein's cosmological constant (Padmanabhan 2003; Frieman, Turner & Huterer 2008; Weinberg 2013), dark energy (Lima 2004; Caldwell & Kamionkowski 2009; Li *et al.* 2011; Frieman, Turner & Huterer 2008; Weinberg 2013) and quantum matter creation from gravitational field (Lima *et al.* 2008; Steigman *et al.* 2009; Lima *et al.* 2010; Graef *et al.* 2014; Jesus & Andrade-Oliveira 2016).

However, this important evidence arising from SNe Ia observations has been questioned through the years and some alternative explanations to the observations have been given. Examples are possible evolutionary effects in SNe Ia events (Drell, Loredó & Wasserman 2000; Combes 2004), local Hubble bubble (Zehavi *et al.* 1998; Conley *et al.* 2007), modified gravity (Ishak, Upadhye & Spergel 2006; Kunz & Sapone 2007; Bertschinger & Zukin 2008), unclustered sources of light attenuation (Aguirre 1999; Rowan-Robinson 2002; Goobar, Bergstrom & Mortsell 2002) and the existence of Axion-Like-Particles (ALPs), arising in a wide range of well-motivated high-energy physics scenarios, and that could lead to the dimming of SNe Ia brightness (Avgoustidis *et al.* 2009, 2010). Nowadays, the dark energy is supported by several independent observational data, such as baryon acoustic oscillations (BAO) and other galaxy clusters observations, cosmic background radiation, observational Hubble constant data as well as age of the Universe (see Frieman, Turner & Huterer 2008 and Weinberg 2013).

With more than 700 Type Ia supernovae discovered (Betoule *et al.* 2014), the constraints on cosmological parameters inferred from SNe Ia are now limited by systematic errors rather than by statistical errors. An important systematic error source is the mapping of the cosmic opacity. The SNe Ia observations are affected by at least four different sources of opacity, namely, the Milky Way, the hosting galaxy, intervening galaxies, and the Intergalactic Medium. The opacity can also occur by extragalactic magnetic fields that can turn photons into unobserved particles (e.g. light axions, chameleons, gravitons, Kaluza-Klein modes) (Avgoustidis *et al.* 2009 & 2010). Recently, an interesting result was obtained by Lima, Cunha & Zanchin (2011). These authors discussed two different scenarios with cosmic absorption and concluded that only if the cosmic

opacity is fully negligible, the description of an accelerating Universe powered by dark energy or some alternative gravity theory must be invoked (see also Li *et al.* 2013).

An interesting way to test the quality of the SNe Ia data has been performed in recent years by confronting them with data sets which are cosmic opacity independent. For instance, Holanda, Carvalho & Alcaniz (2013) as well as Liao, Avgoustidis & Zhengxiang (2015) used current measurements of the expansion rate  $H(z)$  and SNe Ia data to impose cosmological model-independent constraints on cosmic opacity. These authors found that a completely transparent universe is in agreement with the data considered (see also Avgoustidis *et al.* 2009, 2010 for analyses in a flat  $\Lambda$ CDM framework). Holanda & Busti (2014) explored the possible existence of an opacity at higher redshifts ( $z > 2$ ) in  $\Lambda$ CDM context by using  $H(z)$  data and luminosity distance of gamma ray bursts. The samples were compatible with a transparent universe at  $1\sigma$  level.

On the other hand, ambiguous results also have arisen from this kind of analysis. Chen *et al.* (2012) by using baryons acoustic oscillations (BAO) (Eisenstein *et al.* 2005) and SN Ia data found that an opaque universe is preferred in redshift regions  $0.20 - 0.35$ ,  $0.35 - 0.44$  and  $0.60 - 0.73$ , whereas a transparent universe is favoured in redshift regions  $0.106 - 0.20$ ,  $0.44 - 0.57$  and  $0.57 - 0.60$ . When these authors considered the entire redshift range, their result were still consistent with a transparent universe at  $1\sigma$  confidence level (c.l.). Nair, Jhingan & Deepak (2012) also compared distance measurements obtained from SNe Ia and BAO and they found that the supernovae are brighter than expected from BAO measurements. Obtaining a similar result, Basset & Kunz (2004) found a  $2\sigma$  violation caused by excess brightening of SNe Ia at  $z > 0.5$  when confronted them via cosmic distance duality relation with angular diameter distances (ADD) from compact objects, perhaps due to lensing magnification bias. By testing the luminosity distance of SNe Ia with ADD from galaxy clusters, Li *et al.* (2013) have put constraints on cosmic opacity. However, the limits thus obtained rely on the assumptions used to describe the galaxy clusters morphology. The results of Holanda, Carvalho & Alcaniz (2013) also rely on the SNe Ia light curve fitter (SALT2, MLCS2K2). In this way, it is still interesting to investigate and compare different observational data and look for any systematics in them.

In this paper we show that it is possible to obtain opacity free luminosity distances from age estimates of old objects and, by comparing it with luminosity distance from SNe Ia, namely, Union 2.1 compilation, to impose constraints on cosmic opacity. No specific cosmological model is adopted in our analysis. We only consider the flat Friedmann-Robertson-Walker framework and we use the *Planck* constraint on the cosmic total age (Ade *et al.* 2016) to put constraints

over  $t_{inc}$ .

The paper is organized as follows. In Section II we describe our new method to verify the cosmic opacity, while in Section III the observational quantities used in this work are discussed. The corresponding constraints on the opacity are investigated and discussed in Section IV. We summarize our main results in Section V.

## II. METHODOLOGY

### A. Cosmic opacity

As well known, a reduction of photons number from a luminosity source caused by an absorption leads to a modification of its inferred luminosity distance, increasing it with respect to a transparent universe. Mathematically, if there is a opacity or extinction between observer and a light source, the flux received is attenuated by a factor  $e^{-\tau(z)}$  (Chen & Kantowski, 2009a; 2009b). In this way, the inferred luminosity distance of the source,  $D_{L,obs}$  is related to the true luminosity distance (cosmic opacity independent) by

$$D_{L,obs}^2 = D_{L,true}^2 e^{\tau(z)}. \quad (1)$$

Therefore, the observed distance modulus is given by

$$m_{obs} = m_{true} + 2.5[\log e]\tau(z). \quad (2)$$

In our analyses, measurements of  $m_{obs}$  are taken from the SNe Ia Union 2.1 compilation (Suzuki *et al.* 2012). We compare  $m_{obs}$  estimates from SNe Ia data to opacity-free  $m_{true}$  inferred directly from the ages of old objects as will be discussed in the next subsection.

### B. Luminosity distance from old objects

Let us consider the Friedmann-Robertson-Walker context. The comoving distance can be written as follows (Jimenez & Loeb 2002)

$$D_C = c \int_z^0 \left[ (1+z) \frac{dt}{dz} \right] dz, \quad (3)$$

where  $t$  is the Universe age at redshift  $z$ . In this way, if one can access the  $dt/dz$  quantity in SNe Ia redshift range without assumptions on cosmological model and also free of opacity, it is possible to obtain  $m_{true}$  for each SNe Ia redshift and put constraints on  $\tau(z)$  via Eq. (2).

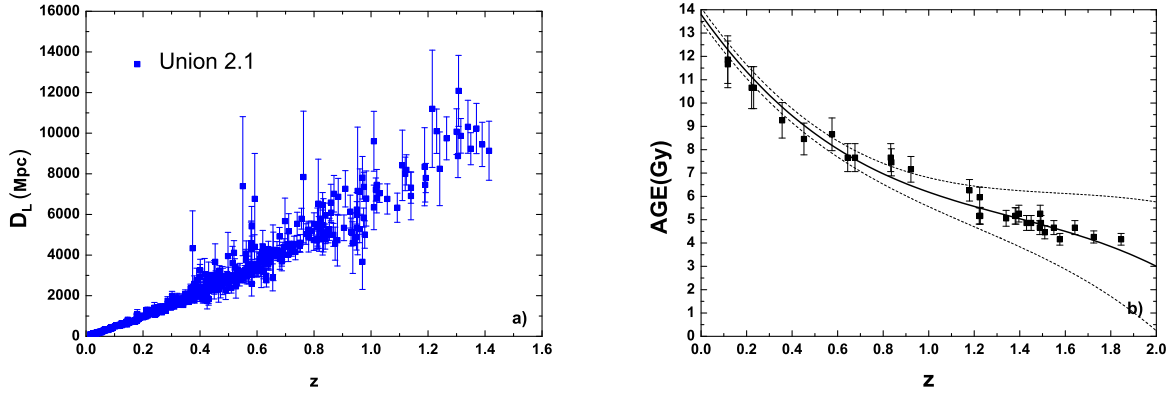


FIG. 1: **(a)** Luminosity distances of SNe Ia from Union 2.1 compilation. **(b)**  $t(z)$  obtained from measurements of the galaxies and the total age from *Planck* results. We plot the original data added with an incubation time  $t_{inc} = 1.66$  Gyr. The solid and dashed black lines are the best fit and  $1\sigma$  error, respectively, obtained from a fit by using a third degree polynomial.

In our work,  $dt/dz$  is estimated from ages of 32 old passive galaxies distributed over the redshift interval  $0.11 < z < 1.84$  (see next section for details). In Fig. 1b we plot the original age added with an incubation time  $t_{inc} = 1.66$  Gyr (see next section for more details). As we are only interested on the derivative  $dt/dz$ , instead of assuming an incubation time and using the total age from other observations, we choose to fit  $t_{obj}(z)$ . If we assume that the incubation time is constant, that is, independent of redshift, one may see that  $t(z)$  only differs from  $t_{obj}(z)$  by a constant. That is,

$$t(z) = t_{obj}(z) + t_{inc}. \quad (4)$$

We have tested some polynomial fits for  $t_{obj}(z)$  and we have found that the minimal polynomial that yields a good fit, when combined with SNs Ia, is a third degree polynomial fit, such as:  $t_{obj}(z) = a_0 + a_1z + a_2z^2 + a_3z^3$ . As we assume  $t_{inc}$  to be constant, we have  $\frac{dt_{obj}}{dz} = \frac{dt}{dz}$ , so we may say, that our model of Universe, that is, the function we assume that can describe the Universe given the data is:

$$\frac{dt}{dz} = a_1 + 2a_2z + 3a_3z^2 \quad (5)$$

From (4), we may also estimate  $t_{inc}$ , once we estimate  $a_0$  and know  $t_0$ , as we have  $t_{inc} = t_0 - a_0$ . Finally, from Eq. (3), the comoving distance can be given by

$$d_c(z) = c \left[ -a_1 \left( z + \frac{z^2}{2} \right) - a_2 \left( z^2 + \frac{2z^3}{3} \right) - a_3 \left( z^3 + 3\frac{z^4}{4} \right) \right]. \quad (6)$$

However, one must note that the parameters  $a_i$  derived from the age of old objects will be in Gyr, so one must multiply (6) by some conversion factor in order to have it in Mpc, for instance.

### III. DATA SET

In the following, we describe the data sets used in our analyses.

- For the cosmic opacity dependent data, we use the Union 2.1 SNe Ia sample. This sample is an update of the original Union compilation (Amanullah *et al.* 2010) that comprises 580 data points including recent large samples from other surveys and uses SALT2 for SNe Ia lightcurve fitting (Guy *et al.* 2007). As the Union 2.1 consists of several subsamples, Suzuki *et al.* (2012) allowed a different absolute magnitude value for each subsample thereby making the impact of the cosmological model negligible.
- As the cosmic opacity independent data, we use the age estimates of 32 old passive galaxies distributed over the redshift interval  $0.11 < z < 1.84$ , as recently analysed by Simon, Verde & Jimenez (2005). The total sample is composed by three sub-samples: 10 field early-type galaxies from Treu *et al.* (1999, 2001, 2002), whose ages were obtained by using SPEED models of Jimenez *et al.* (2004); 20 red galaxies from the publicly released Gemini Deep Deep Survey (GDDS) whose integrated light is fully dominated by evolved stars (Abraham *et al.* 2004, McCarthy *et al.* 2004). Simon, Verde & Jimenez (2005) re-analysed the GDDS old sample by using a different stellar population models and obtained ages within 0.1 Gyr of the GDDS collaboration estimates - and the two radio galaxies LBDS 53W091 and LBDS 53W069 (Dunlop *et al.* 1996; Spinrad *et al.* 1997; Nolan *et al.* 2001).

Recently, Wei *et al.* (2015) obtained  $t_{inc} = 1.36$  Gyr and  $t_{inc} = 1.62$  Gyr by considering this galaxy sample in a flat  $\Lambda$ CDM model with fixed and free  $H_0$  (Hubble parameter) and  $\Omega_m$  parameters, respectively. This factor accounts for our ignorance about the amount of time since the beginning of the structure formation in the Universe until the formation time of the object. However, such a treatment assumes that all of these galaxies need to have formed at the same time for their ages to trace out the Universe history. We still assume 10% uncertainty on measurement of age of each Galaxy (Dantas *et al.* 2009, 2011; Samushia *et al.* 2010). As commented by Avgoustidis *et al.* (2009, 2010) the method of age determination relies on the detailed shapes of galaxy spectra but not on galaxy luminosities. It will therefore not be affected by a non-zero opacity since  $\tau$  is assumed (and

constrained by independent observations) not to be strongly wavelength dependent in the optical band (see also Mortsell & Goobar 2003a, 2003b).

#### IV. ANALYSES AND DISCUSSION

We estimate the best-fit to the set of parameters  $\mathbf{p} \equiv (\epsilon, a_0, a_1, a_2, a_3)$ , by evaluating the likelihood distribution function,  $\mathcal{L} \propto e^{-\chi^2/2}$ , with

$$\chi^2(\mathbf{p}) = \chi_{tz}^2(a_0, a_1, a_2, a_3) + \chi_{SN}^2(\epsilon, a_1, a_2, a_3) \quad (7)$$

where

$$\chi_{tz}^2 = \sum_{i=1}^{32} \frac{(t_{obj,obs,i} - t_{obj}(z_i))^2}{\sigma_{t_{obj,obs,i}}^2} \quad (8)$$

and

$$\chi_{SN}^2 = \sum_{i=1}^{580} \frac{(m_{obs,i} - m_{true}(z_i) - 1.085736\tau(z_i))^2}{\sigma_{m,obs,i}^2}, \quad (9)$$

where  $m_{true} = 5 \log_{10} d_{L,true} + 25$  and  $\sigma_{m(obs)}$  is the distance modulus and its uncertainty from SNe Ia. The cosmic opacity,  $\tau(z)$ , is parametrized by two functions:

- P1 -  $\tau(z) = 2\epsilon z$ . This linear expression can be derived from the usual cosmic distance duality relation parametrization  $D_L = D_A(1+z)^{2+\epsilon}$  for small values of  $\epsilon$  and  $z \leq 1$ , where  $\epsilon$  quantifies departures from transparency (see Avgoustidis *et al.* 2009).
- P2 -  $\tau(z) = \epsilon z/(1+z)$ , which avoids the  $\tau(z)$  divergence at high redshifts of the linear parametrization.

Because there is so many free parameters in both models, we choose to sample the likelihood through Monte Carlo Markov Chain (MCMC) analysis. A simple and powerful MCMC method is the so called Affine Invariant MCMC Ensemble Sampler by Goodman and Weare (2010), which was implemented in Python language with the `emcee` software by Foreman-Mackey *et al.* (2013). This MCMC method has the advantage over simple Metropolis-Hasting (MH) methods of depending on only one scale parameter of the proposal distribution and on the number of walkers. While MH methods in general depend on the parameter covariance matrix, that is, it depends on  $n(n+1)/2$  tuning parameters, where  $n$  is dimension of parameter space. The main idea of the Goodman-Weare affine-invariant sampler is the so called “stretch move”, where the position (parameter vector in parameter space) of a walker (chain) is determined by the position

Parameter	P1	P2
$\epsilon$	$0.016^{+0.038+0.078}_{-0.038-0.075}$	$-0.18^{+0.12+0.25}_{-0.13-0.24}$
$a_0$ (Gyr)	$12.14 \pm 0.29 \pm 0.58$	$12.57^{+0.27+0.54}_{-0.27-0.53}$
$a_1$ (Gyr)	$-13.778^{+0.064+0.13}_{-0.065-0.13}$	$-13.845^{+0.077+0.15}_{-0.078-0.15}$
$a_2$ (Gyr)	$8.16^{+0.49+0.96}_{-0.48-0.96}$	$7.43^{+0.44+0.86}_{-0.44-0.87}$
$a_3$ (Gyr)	$-1.98^{+0.24+0.48}_{-0.24-0.47}$	$-1.61 \pm 0.23 \pm 0.46$
$\chi^2_{red}$	0.981	0.978

TABLE I: Marginalized results for the free parameters of models P1 and P2. The values shown correspond to mean values of the parameters. The best fit values are much similar, as the distributions are quite symmetrical.

of the other walkers. Foreman-Mackey *et al.* modified this method, in order to make it suitable for parallelization, by splitting the walkers in two groups, then the position of a walker in one group is determined by *only* the position of walkers of the other group<sup>1</sup>.

We used the freely available software `emcee` to sample from our likelihood in our 5-dimensional parameter space. We have used flat priors over the parameters. In order to plot all the constraints in the same figure, we have used the freely available software `getdist`<sup>2</sup>, in its Python version. The results of our statistical analyses from Eq. (6) can be seen in Fig. 2 and Table I.

From Fig. 2 and Table I, we see that both functions (P1,P2) favor a transparent universe at least into  $2\sigma$  c.l.. In fact, the main information we may extract from Table I is that we have found the superior limits  $\epsilon < 0.094$  for P1 and  $\epsilon < 0.07$  for P2, both limits at 95% c.l. Figure 3a shows the marginalized likelihoods for  $\epsilon$  in both models. An interesting result appears when the evolution of  $\tau(z)$  is plotted. As one may see in Fig. 3b, the transparent universe is in full agreement with the data used in our analyses into  $1\sigma$  c.l. for model P1 and  $2\sigma$  c.l. for model P2.

As we have claimed before, we may estimate incubation time in context of our models once we have estimated  $a_0$  and by taking the total age from Planck (Ade *et al.*, 2016). The total age indicated by Ade *et al.* in their TT+lowP+lensing analysis, in the context of flat  $\Lambda$ CDM model, was  $t_0 = 13.799 \pm 0.038$  Gyr. As  $t_{inc} = t_0 - a_0$ , we have  $t_{inc} = 1.66 \pm 0.29$  Gyr for model P1 and  $t_{inc} = 1.23 \pm 0.27$  Gyr for model P2, both at 68% c.l. These values are in agreement with the ones obtained by Wei *et al.* (2015), in the context of flat  $\Lambda$ CDM model, where they have found  $t_{inc} = 1.36$  Gyr for fixed and  $t_{inc} = 1.62$  Gyr for free  $H_0$  and  $\Omega_m$  parameters.

<sup>1</sup> See Allison and Dunkley (2013) for a comparison among various MCMC sampling techniques.

<sup>2</sup> `getdist` is part of the great MCMC sampler and CMB power spectrum solver `COSMOMC`, by Lewis and Bridle (2002).



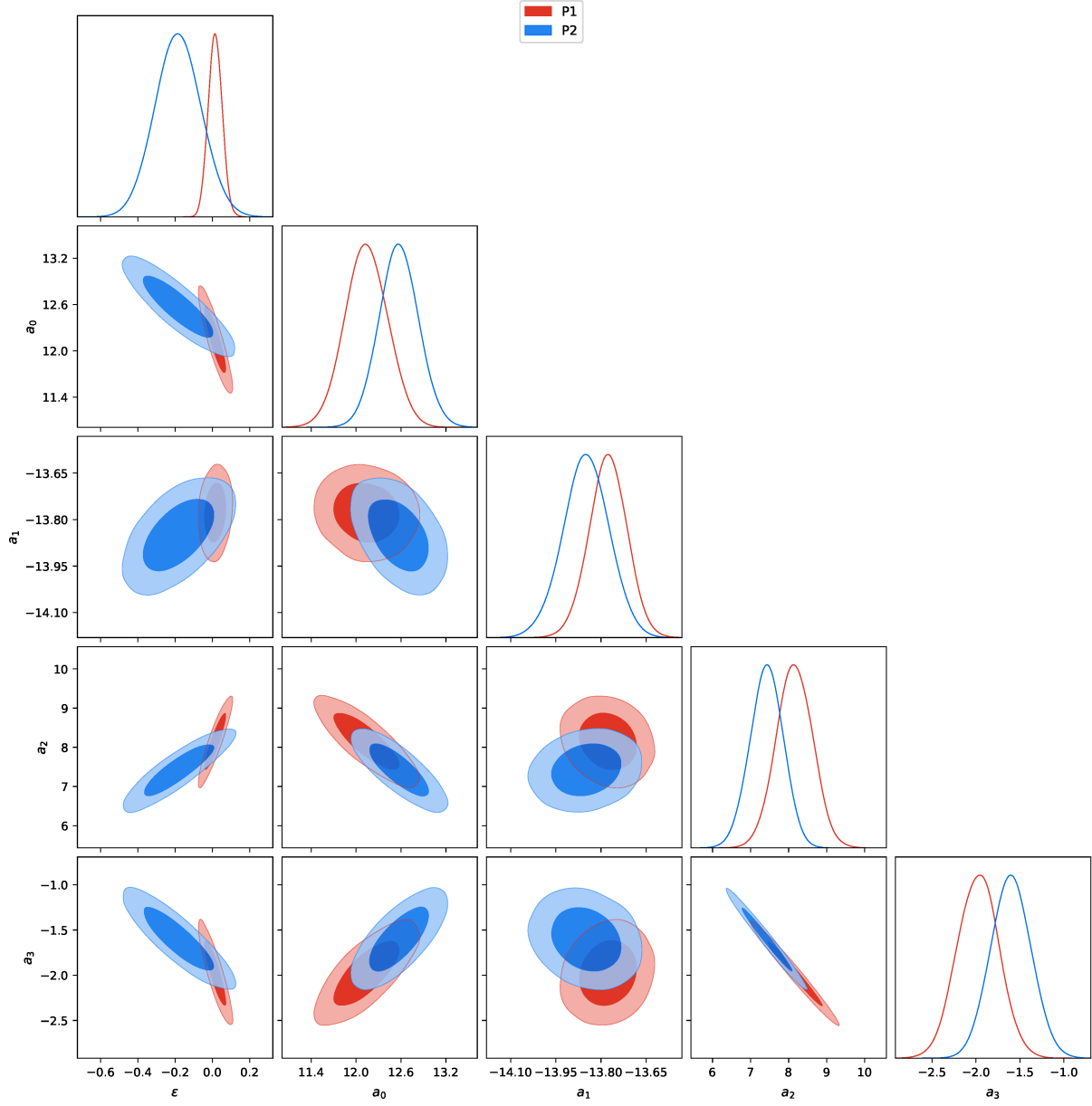


FIG. 2: The results of our statistical analysis, with constraints from SNs Union 2.1 data and ages of old objects. **Diagonal:** Likelihoods for the parameters on each indicated model,  $P_1$  (red) and  $P_2$  (blue). **Below diagonal:** Contours for 68.3% and 95.4% confidence intervals for each indicated model,  $P_1$  and  $P_2$ . The  $a_i$  parameters have units of Gyr. Although we represent the  $\epsilon$  parameter of each model in the same column, they have different meanings, because they correspond to different  $\tau_i(z)$ .

### A. Comparing results

At this point it is interesting to compare our results with previous ones that used the linear parametrization and different observations. For instance, Avgoustidis *et al.* (2009, 2010) via SNe

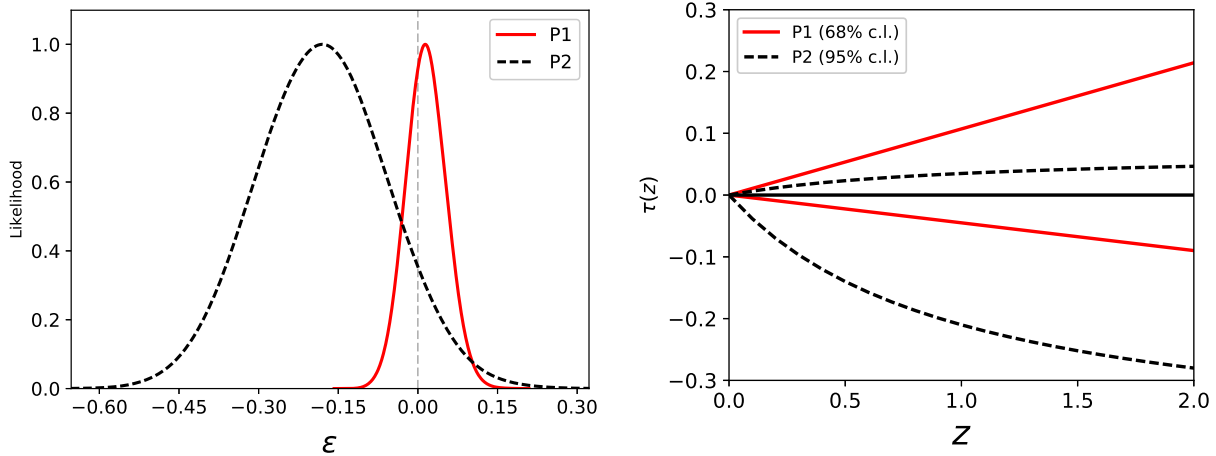


FIG. 3: (a) Likelihood functions for  $\epsilon$ . (b) Confidence intervals on the plane  $\tau - z$ , 68% c.l. for P1 and 95% c.l. for P2. In both figures the solid and dashed lines correspond to results from P1 and P2.

Ia and  $H(z)$  data obtained  $\epsilon = -0.01^{+0.08}_{-0.09}$  and  $\epsilon = -0.04^{+0.08}_{-0.07}$  in the flat  $\Lambda$ CDM framework. Holanda, Carvalho & Alcaniz (2013) by using only SNe Ia +  $H(z)$  observations obtained  $\epsilon = 0.017 \pm 0.055$ . Holanda & Busti (2014) by using gamma ray bursts +  $H(z)$  observations obtained  $\epsilon = 0.03 \pm 0.10$  and  $\epsilon = 0.028 \pm 0.10$  in flat  $\Lambda$ CDM and XCDM frameworks, respectively. More recently, Liao, Avgoustidis & Li (2015) also used only SNe Ia +  $H(z)$  data, but they have taken into account the covariance between the distances from  $H(z)$  measurements obtained from integration on the  $H(z)$  data, found  $\epsilon = 0.07^{+0.11}_{-0.12}$ , in full agreement with our results. None of these analyses have been able to discard a transparent Universe.

## V. CONCLUSIONS

In this paper we have proposed a new model cosmological independent method to probe the cosmic opacity. Although that the Universe acceleration is supported by several other independent probes, investigating the cosmic opacity on the SNe Ia data is an important issue, in order to search for some source of unknown systematic error. If some extra dimming is still present, the SNe Ia observations will give us unreal values to main cosmological parameters and the Universe will seem as accelerating at a different rate than it actually is.

To perform our analyses, we have considered the following cosmological data: 580 SNe Ia from Union 2.1 compilation and old objects, specifically, 32 old galaxies ( $0.11 < z < 1.84$ ). Since the method to determine the ages relies on the detailed shapes of galaxy spectra but not on luminosities, they are cosmic opacity independent. We have shown the possibility of obtaining

opacity free luminosity distance from the link, in a flat FRW framework, between  $D_L$  and  $dt/dz$  quantity from a best fit polynomial to  $t(z)$  of old objects. Our ignorance about a possible evolution of opacity was parametrized by  $\tau(z) = 2\epsilon z$  (P1) and  $\tau(z) = \epsilon z/(1+z)$  (P2) and we have found that  $\epsilon$  is compatible with 0 at  $1\sigma$  c.l. for model P1 and at  $2\sigma$  c.l. for model P2 (see Fig. 3). Thus, our results have reinforced the transparency of the universe along with other analyses made in the literature, where were used SNe Ia, angular diameter distance and  $H(z)$  data as well as have reinforced the present accelerated stage of the Universe.

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